スパースモデリングの電波天文学への展開

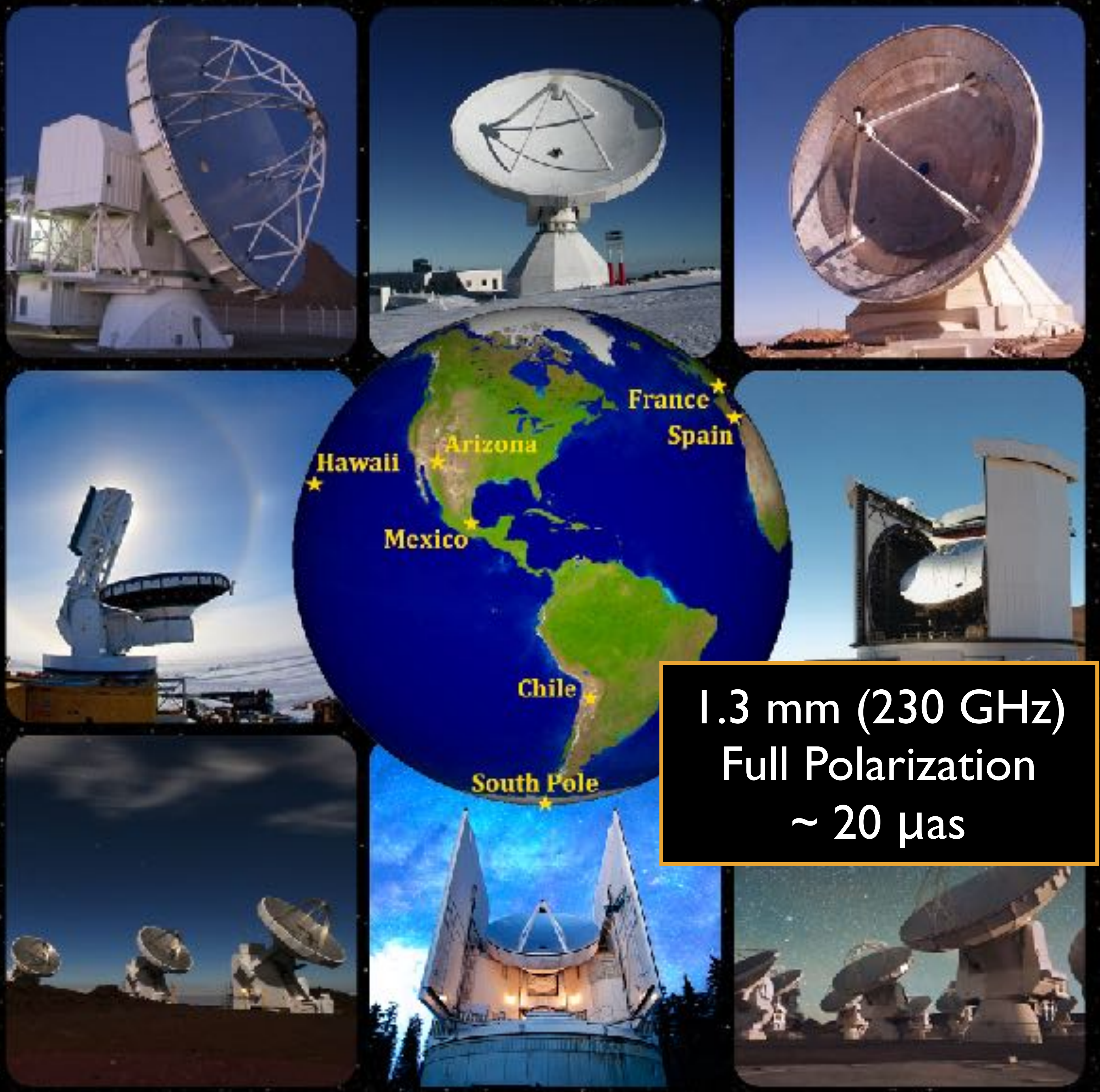
— 超解像多次元イメージングから磁場トモグラフィーまで —

秋山 和徳 (Kazu Akiyama)

(NRAO Jansky Fellow / MIT Haystack Observatory)

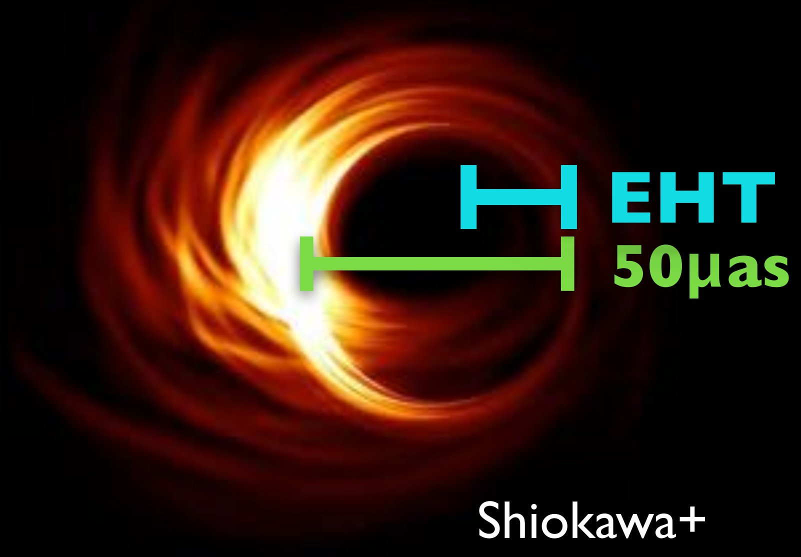
The Event Horizon Telescope Consortium

Black Hole Imaging with the Event Horizon Telescope



1.3 mm (230 GHz)
Full Polarization
~ 20 μas

Sgr A*



Shiokawa+

M87

Moscibrodkza, Dexter+17

Interferometry:
Sampling Fourier Components of the Images

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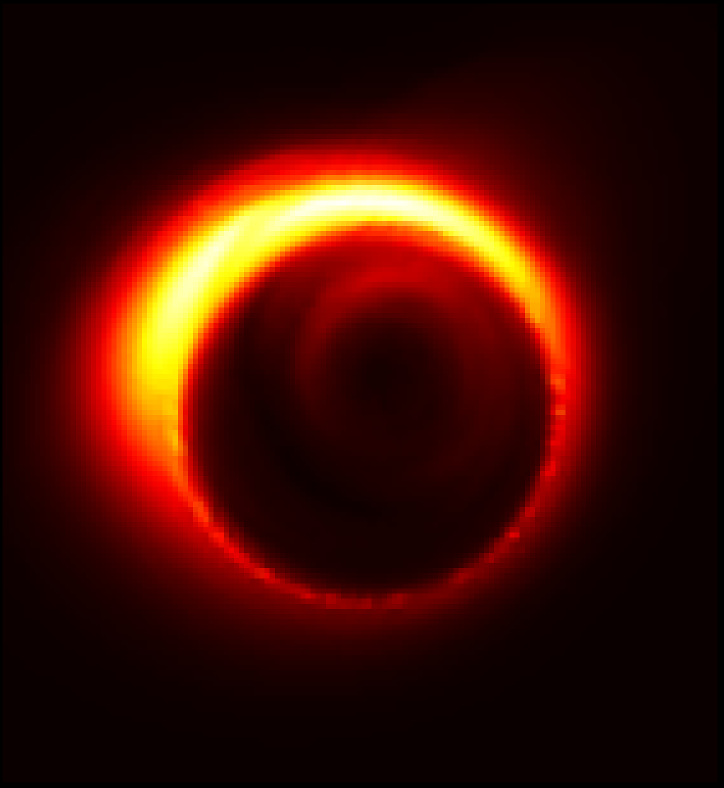
Image



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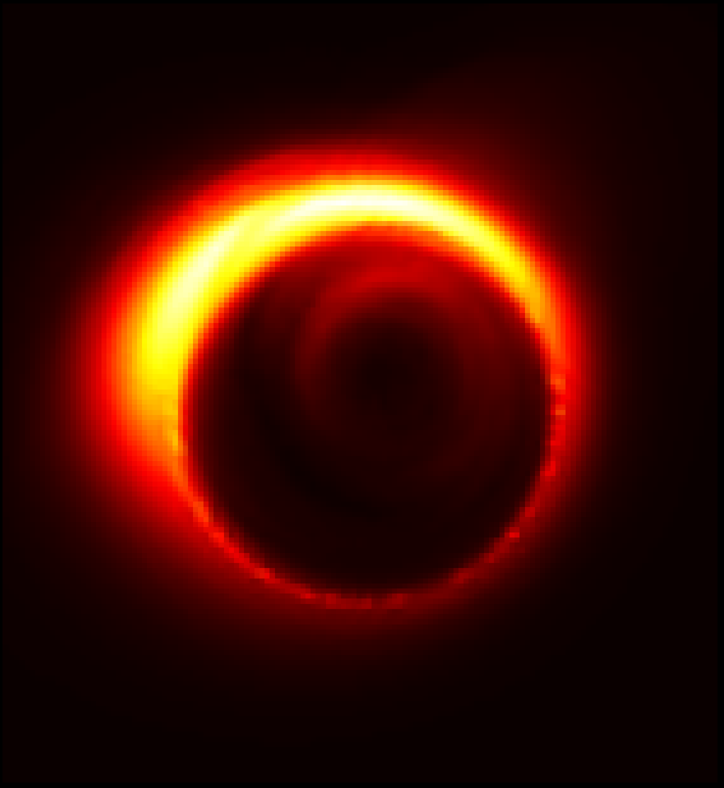
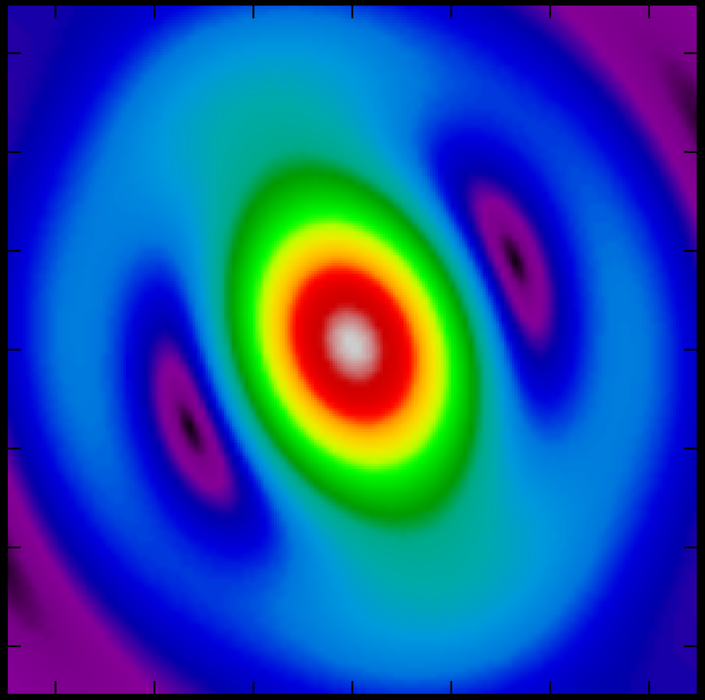


Fourier Domain

(Visibility)



Event Horizon Telescope

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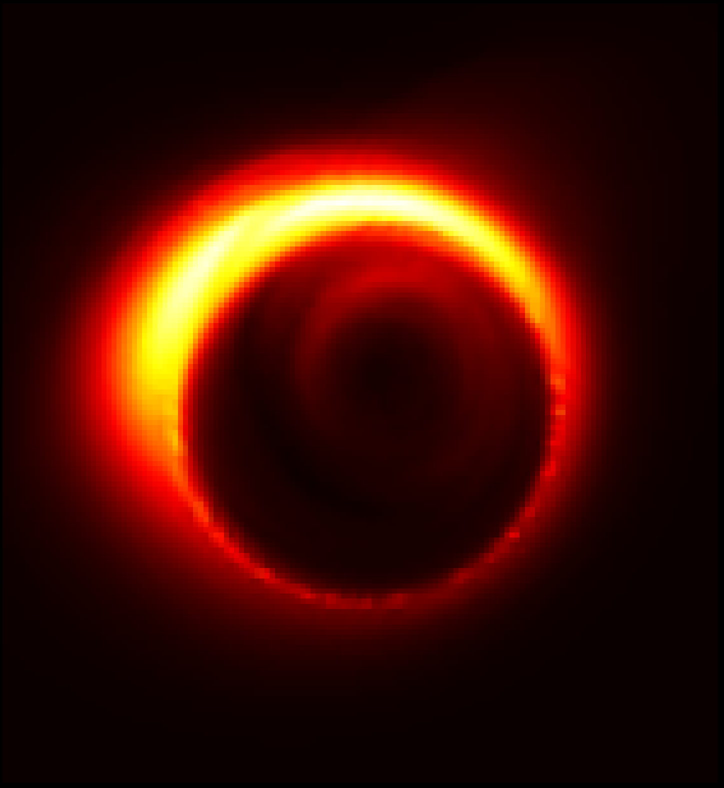

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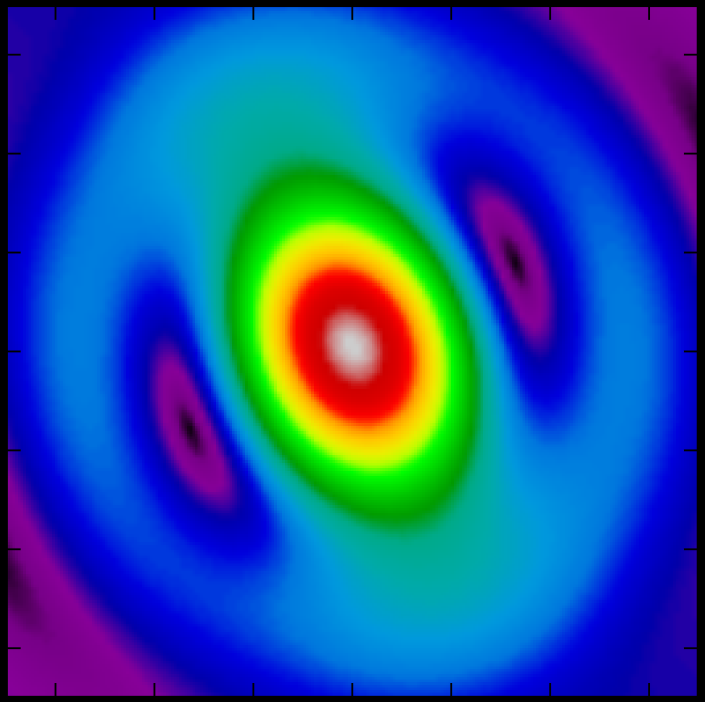
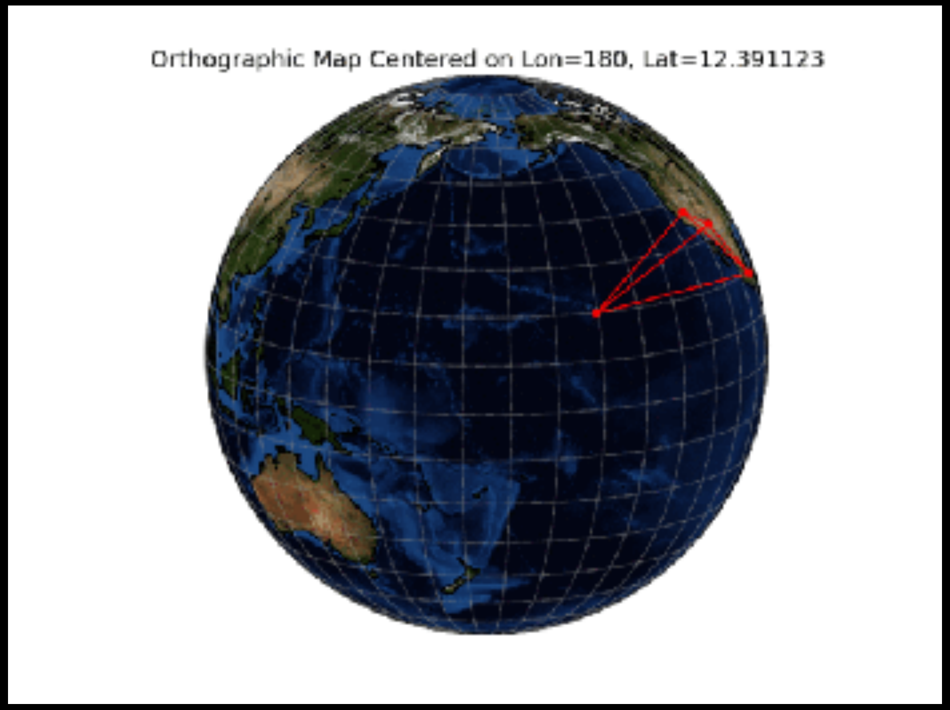


Sampling Process
(Projected Baseline = Spatial Frequency)



(Images: adapted from *Akiyama* et al. 2015, ApJ ; Movie: Laura Vertatschitsch)

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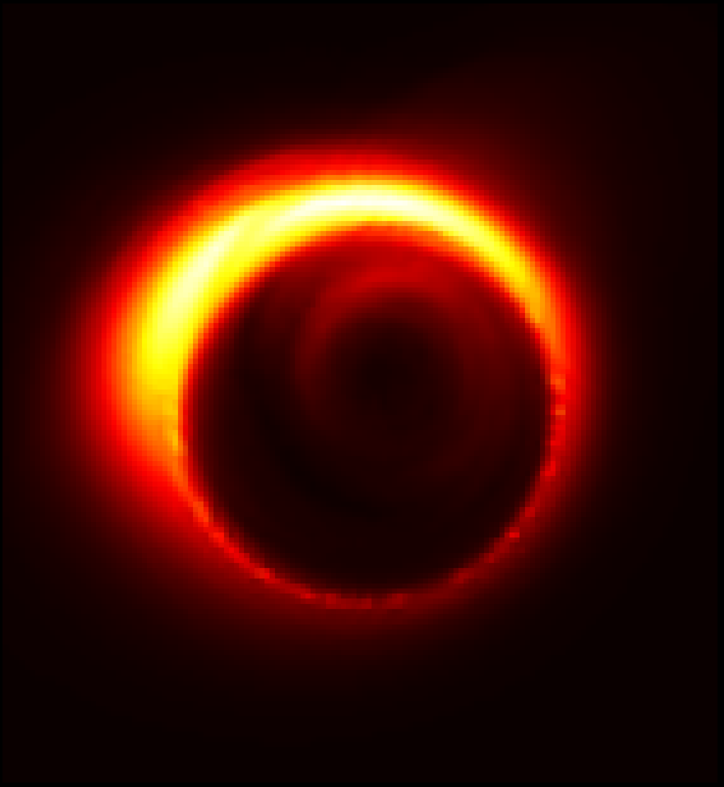

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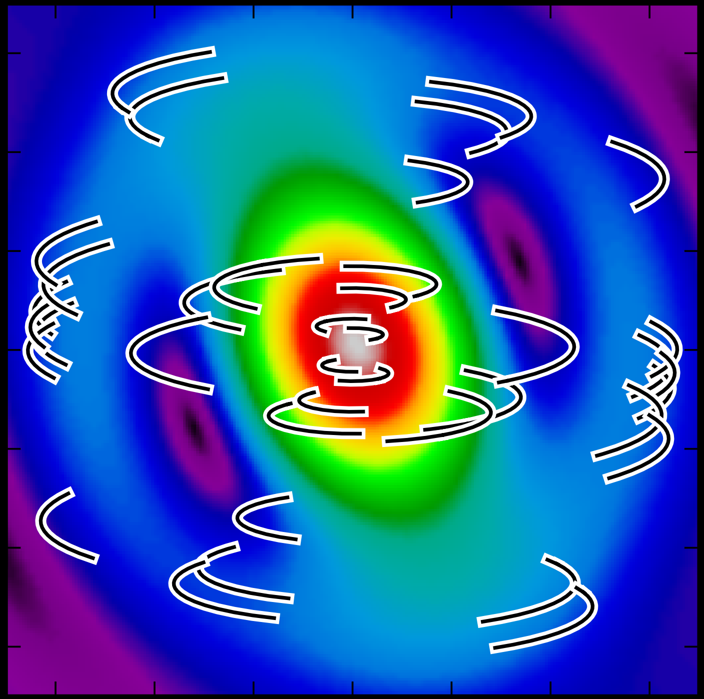
(*Visibility*)



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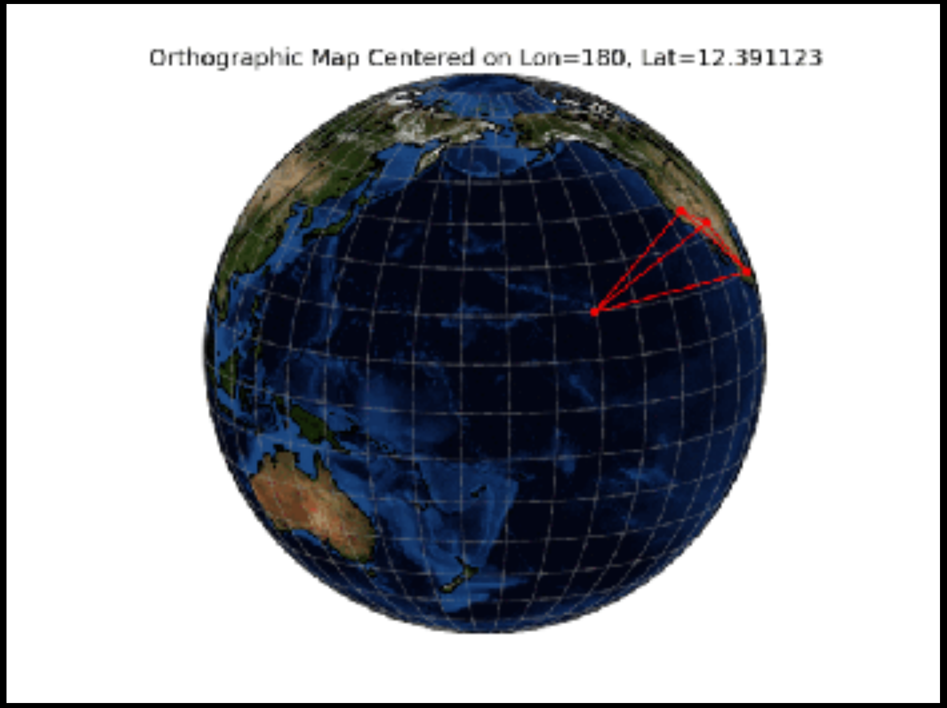

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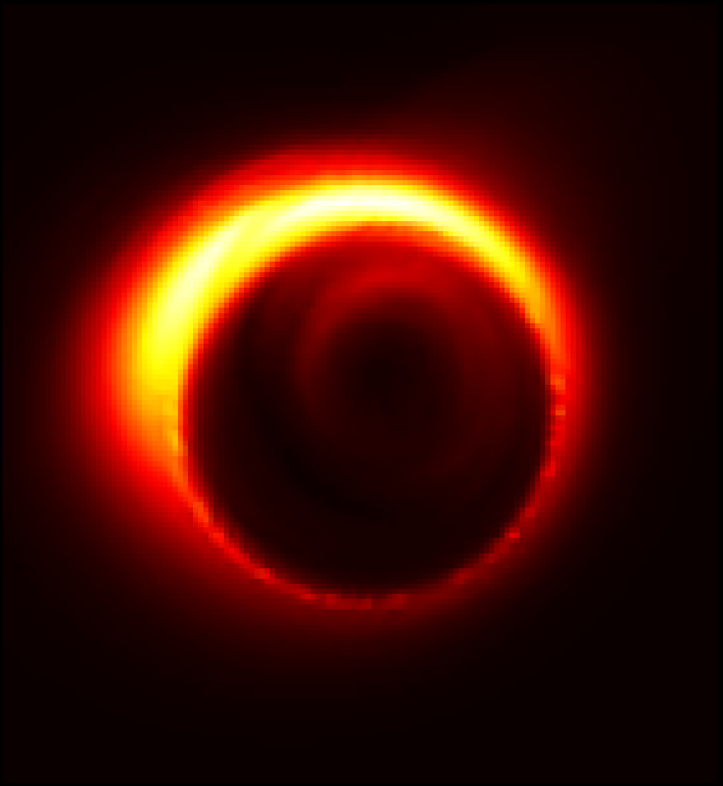

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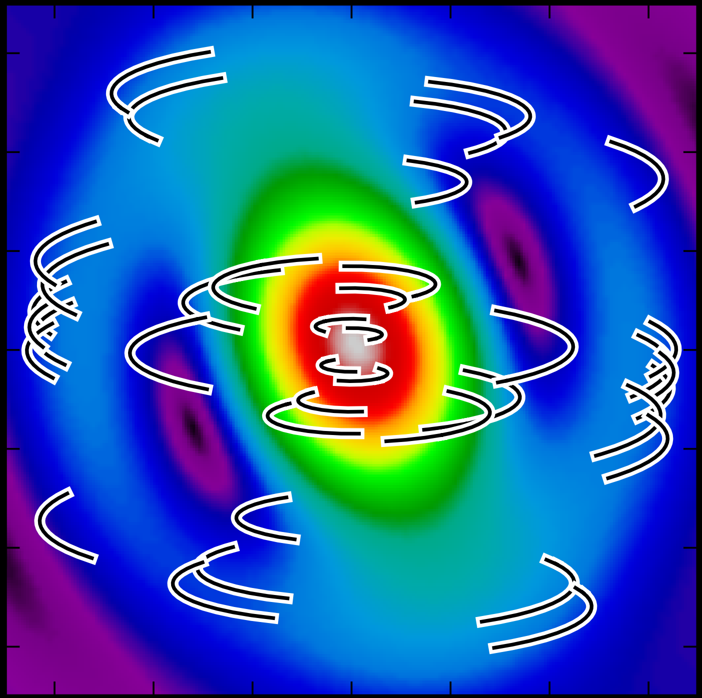

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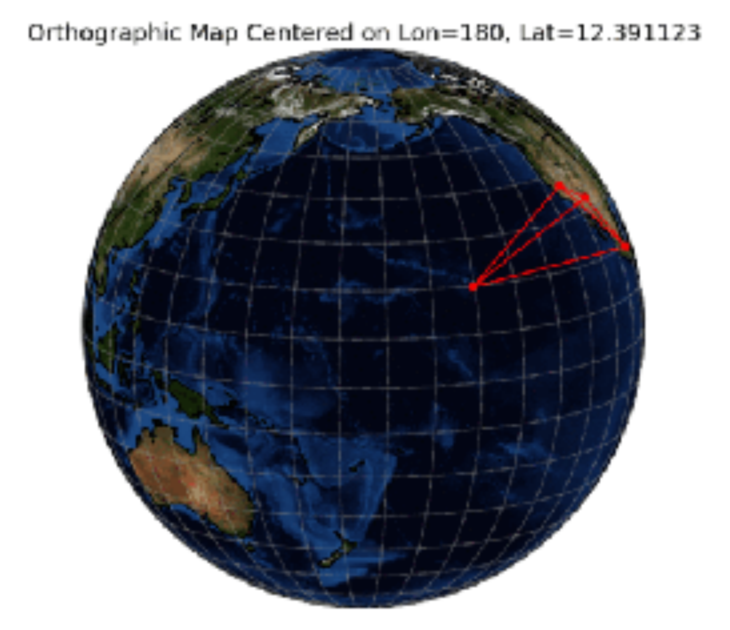

(Images: adapted from *Akiyama* et al. 2015, ApJ ; Movie: Laura Vertatschitsch)

Sampling is NOT perfect

Interferometric Imaging: Observational equation is *ill-posed*

$$\begin{array}{cccc} V & = & A & I \\ \text{(Data)} & & \text{(Fourier Matrix)} & \text{(Image)} \end{array}$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_M \end{pmatrix} = \begin{pmatrix} \exp(i2\pi u_1 x_1) & \exp(i2\pi u_1 x_2) & \ldots & \exp(i2\pi u_1 x_N) \\ \exp(i2\pi u_2 x_1) & \exp(i2\pi u_2 x_2) & \ldots & \exp(i2\pi u_2 x_N) \\ \exp(i2\pi u_3 x_1) & \exp(i2\pi u_3 x_2) & \ldots & \exp(i2\pi u_3 x_N) \\ \vdots & \vdots & & \vdots \\ \exp(i2\pi u_M x_1) & \exp(i2\pi u_M x_2) & \ldots & \exp(i2\pi u_M x_N) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{pmatrix}$$

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  Number of data ***M*** < Number of image pixels ***N***

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V = A I

(Data) (Fourier Matrix) (Image)

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- Equation is *ill-posed*: infinite numbers of solutions

Event Horizon Telescope

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- Sampling is NOT perfect
  Number of data ***M*** < Number of image pixels ***N***
- Equation is *ill-posed*: infinite numbers of solutions
- Interferometric Imaging:
  Picking a reasonable solution based on a prior assumption

Sparse Reconstruction: A Popular Approach

Event Horizon Telescope

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Philosophy: **Reconstructing images with the smallest number of point sources within a given residual error**

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$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \ \text{subject to} \ ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 < \varepsilon$$

L_p-norm:

$$||\mathbf{x}||_p = \left(\sum_i |x_i|^p\right)^{\frac{1}{p}} \quad (p>0)$$

$||x||_0$ = number of non-zero pixels in the image

Event Horizon Telescope

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Number of non-zero pixels (point sources)

Data

Obs. Matrix

Image

Chi-square: Consistency between data and the image

L$_p$-norm:

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Computationally very expensive!!
(It can be solved for N < ~100)

- L_0 norm is not continuous, nondifferentiable
- Combinational Optimization

$||x||_0$ = number of non-zero pixels in the image

HAYSTACK OBSERVATORY

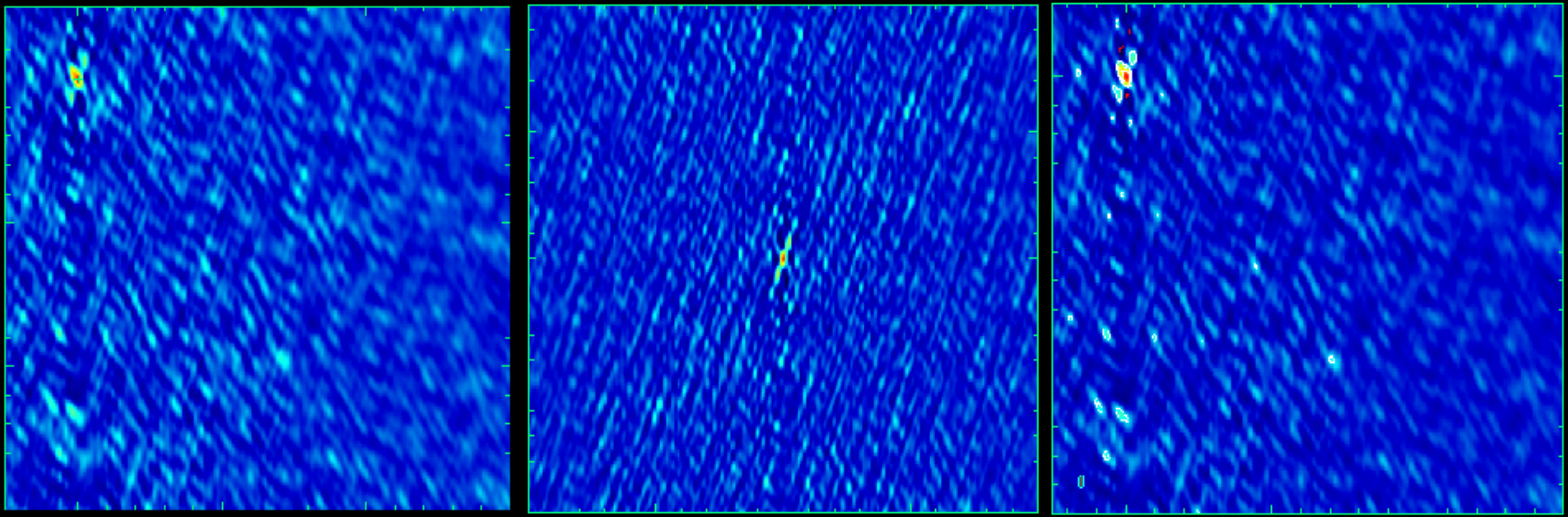
NRAO

Event Horizon Telescope

Sparse Reconstruction: CLEAN (greedy approach)

CLEAN (Hobgom 1974) = Matching Pursuit (Mallet & Zhang 1993)

Computationally very cheap, but highly affected by the Point Spread Function



Dirty map:
FT of *zero-filled*
Visibility

Point Spread Function:
Dirty map
for the point source

Solution:
Point sources
+ Residual Map

(3C 273, VLBA-MOJAVE data at 15 GHz)

Event Horizon Telescope

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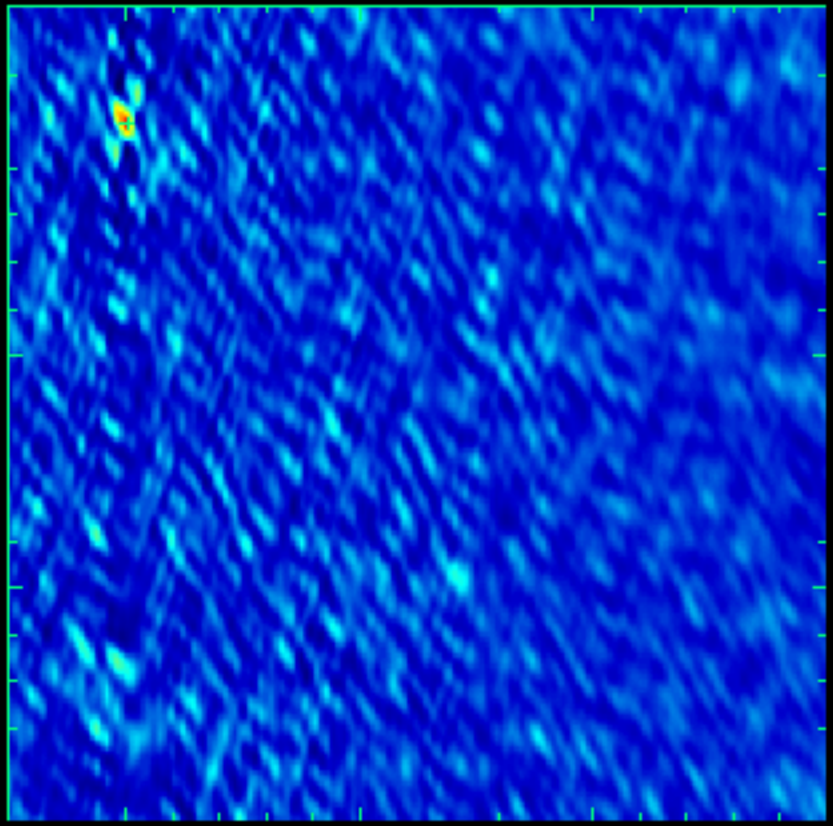

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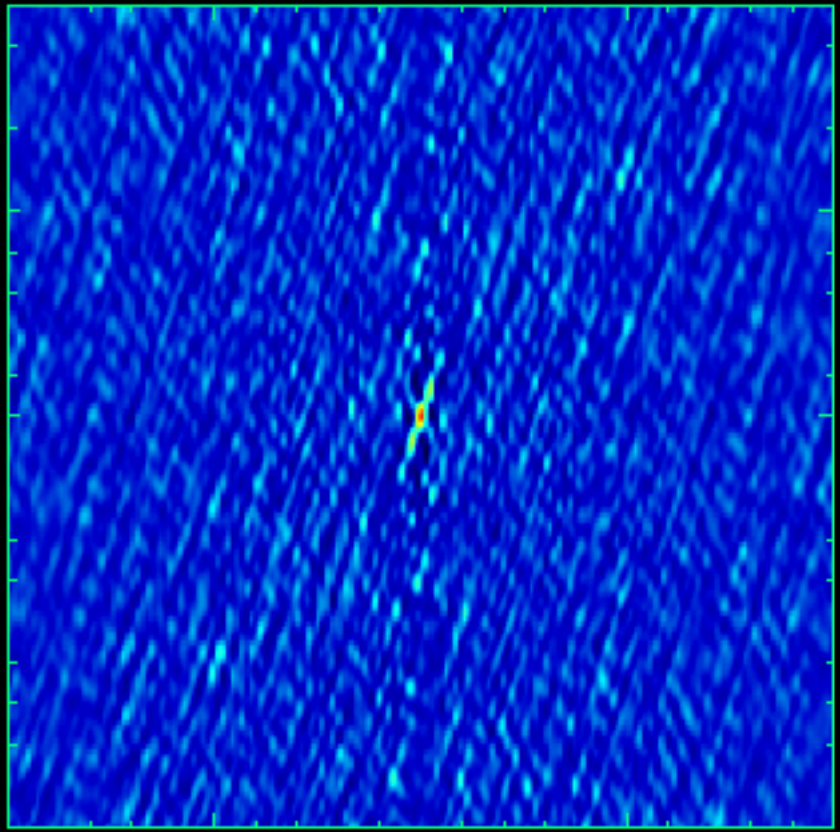

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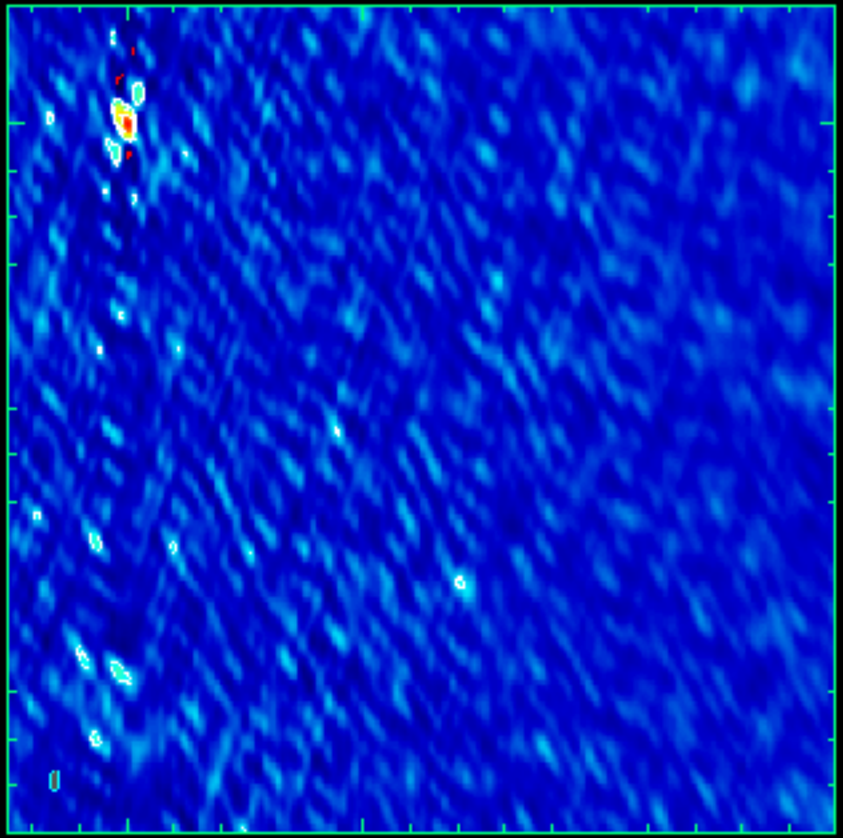

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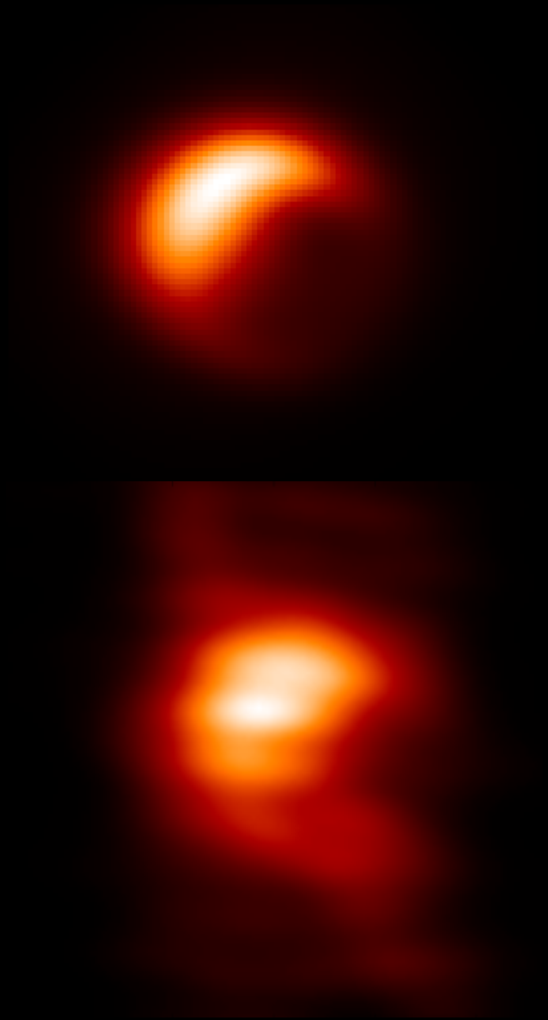
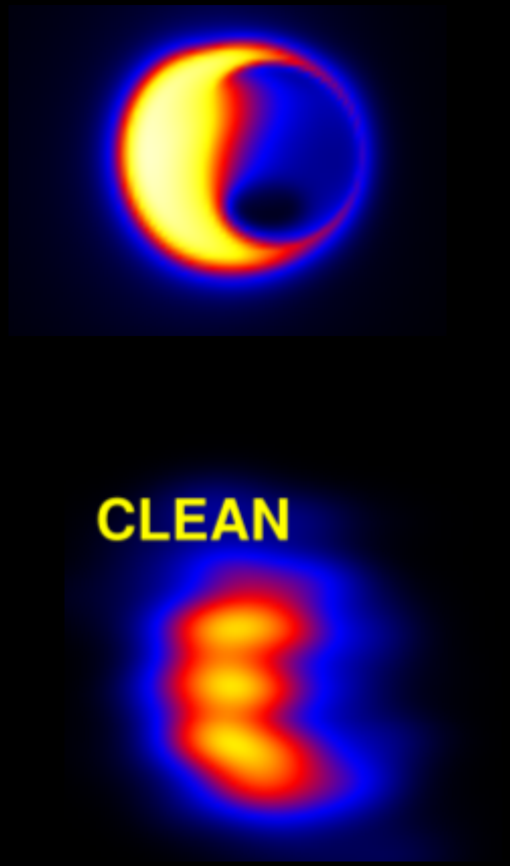
CLEAN is problematic for the black hole shadows?

Ground Truth

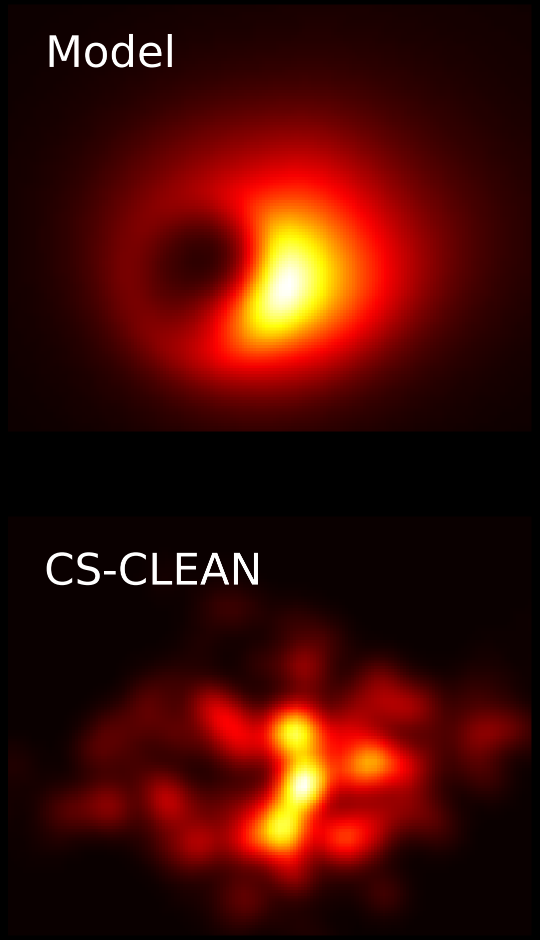
CLEAN



Fabian Baron, EHT 2012



Chael+2016 ApJ



Akiyama+2017a, ApJ
Akiyama+2017b, AJ

Sparse Reconstruction: L1 Regularization

LASSO (Tibishirani 1996)

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Convex Relaxation: Relaxing L0-norm to a convex, continuous, and differentiable function

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⟷ equivalent

$$\min_{\mathbf{x}} \left(\underbrace{||\mathbf{y}-\mathbf{A}\mathbf{x}||_2^2}_{\text{Chi-square}} + \underbrace{\Lambda_l ||\mathbf{x}||_1}_{\text{Regularization on } \textit{sparsity}}\right).$$

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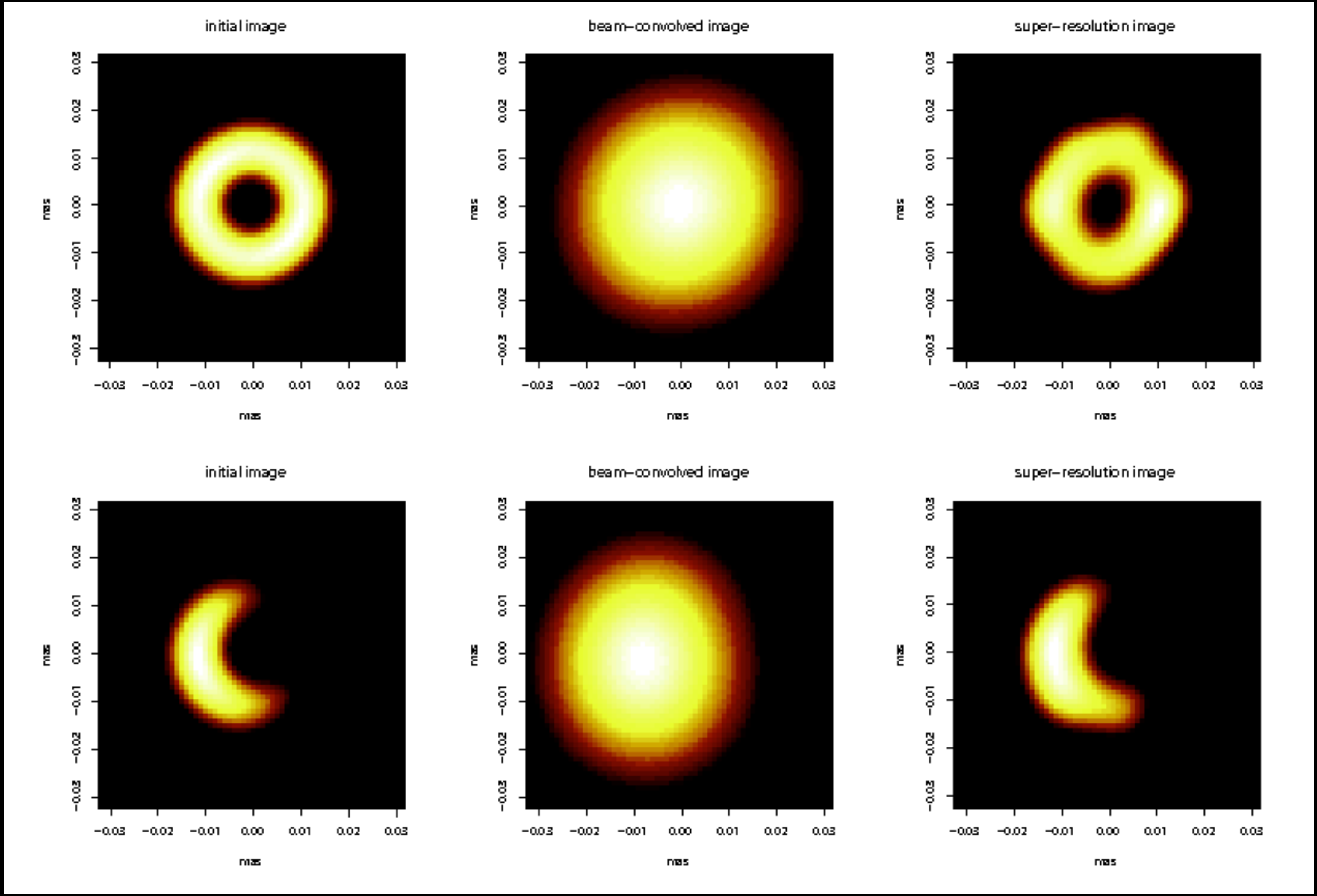
- Reconstruction purely in the visibility domain:
    Not affected by de-convolution beam (point spread function)
  Many applications after appearance of *Compressed Sensing* (Donoho, Candes+)

Sparse Reconstruction: L1 Regularization

LASSO (Tibishirani 1996)

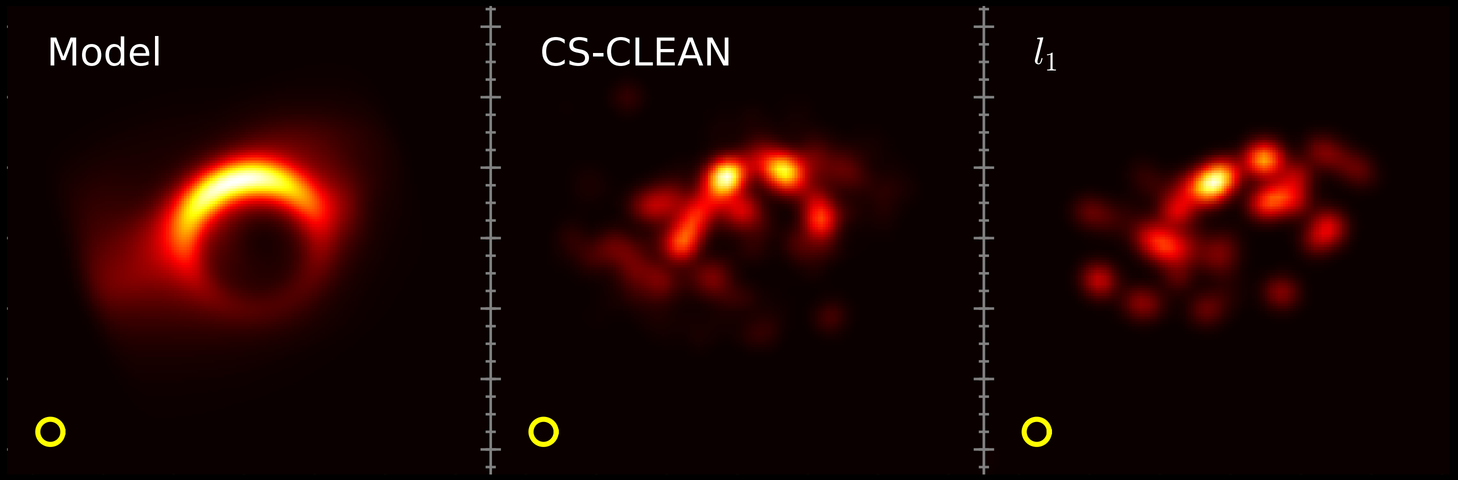


(Honma, *Akiyama*, Uemura & Ikeda 2014, PASJ)

Event Horizon Telescope

Pursuing only sparsity is not optimal

A key assumption in CLEAN and L1 regularization: images must be sparse.



May NOT work!

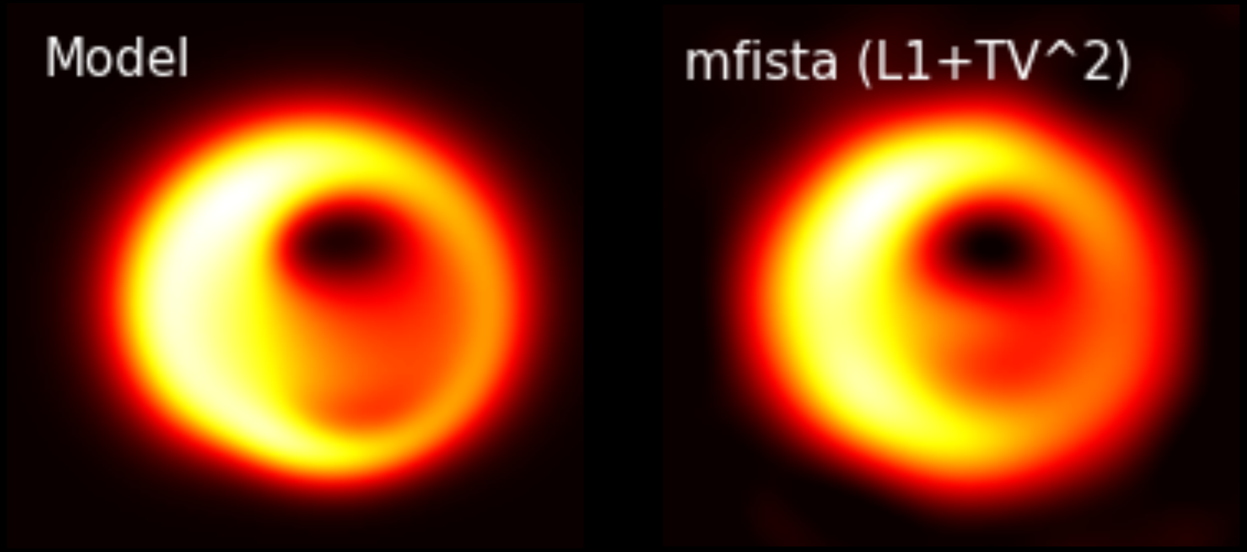
- Extended source
- Even compact source with too small image pixels

Akiyama et al. 2017b, AJ

We need somewhat sparse and smooth images NOT depending on adopted sizes of imaging pixels.

Sparse Modeling on the Gradient Image

$$\min_{\mathbf{x}} \left(||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 + \Lambda_l ||\mathbf{x}||_1 + \Lambda_t ||\mathbf{x}||_{\mathrm{tv}} \right)$$



Kuramochi & *KA* et al. 2018
ApJ, in press

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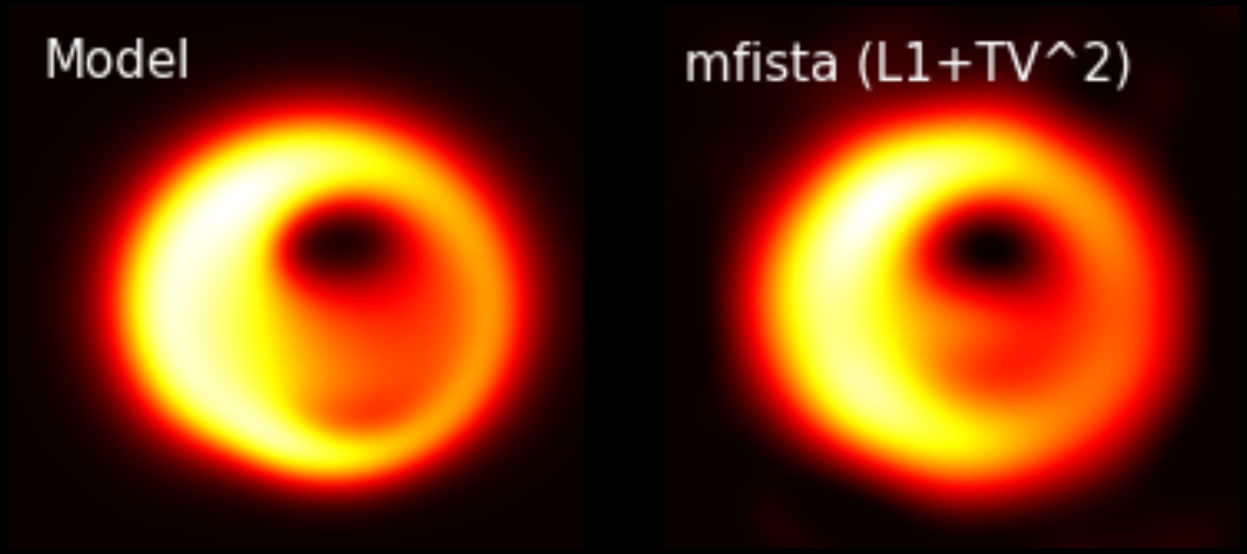

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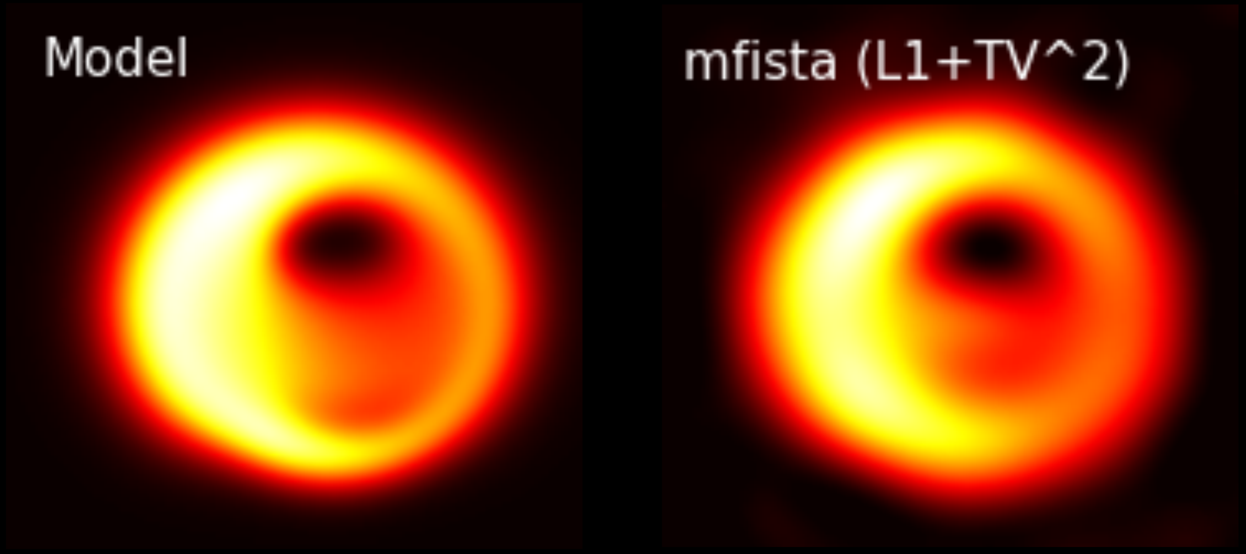

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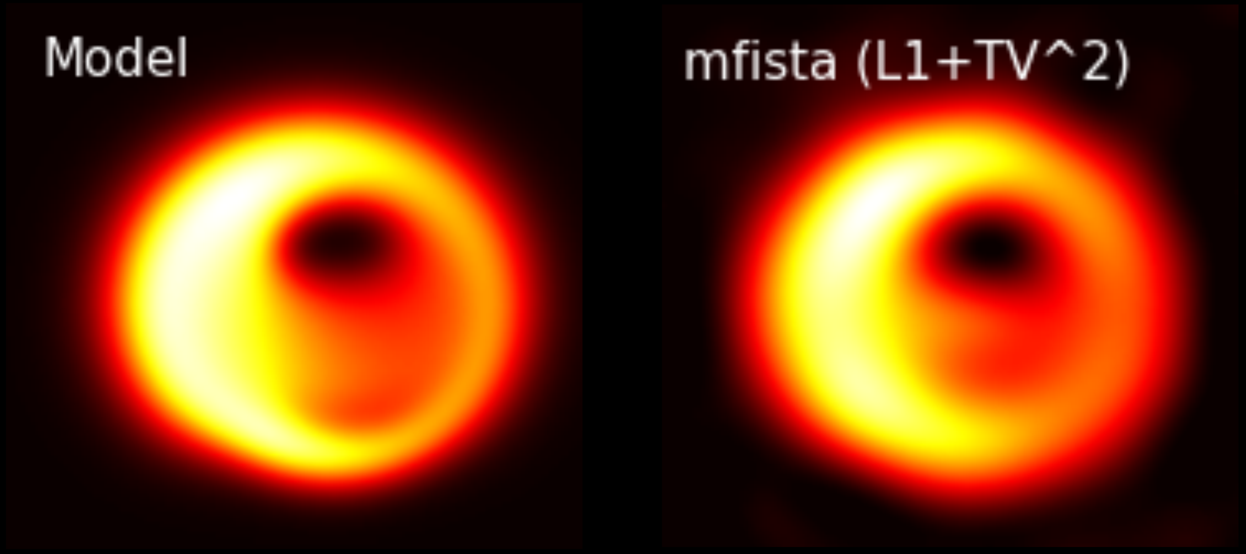
Regularizing the *sparsity on the gradient domain*
= Favoring smooth images



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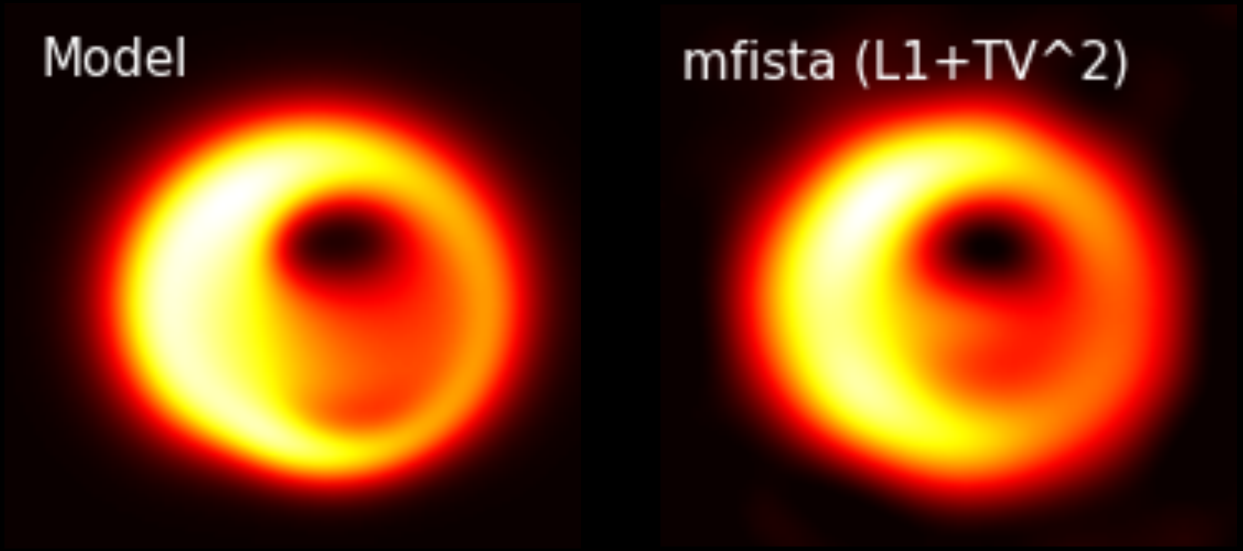
Regularizing the *sparsity on the gradient domain*
= Favoring smooth images

$$||\mathbf{x}||_{\rm tv} = \sum_i \sum_j \left(|x_{i+1,j} - x_{i,j}|^2 + |x_{i,j+1} - x_{i,j}|^2 \right) .$$



Kuramochi & *KA* et al. 2018
ApJ, in press

Application to Real Data: Protoplanetary Disks

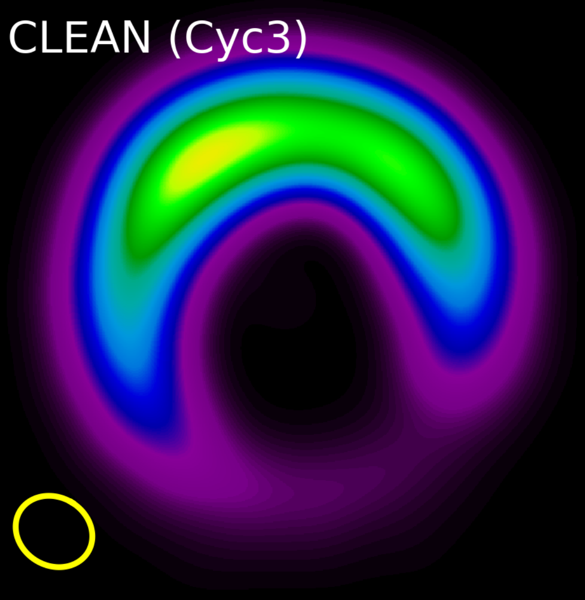
ALMA Observations of Protoplanetary Disk HD 142527 (345 GHz)

Compact configuration

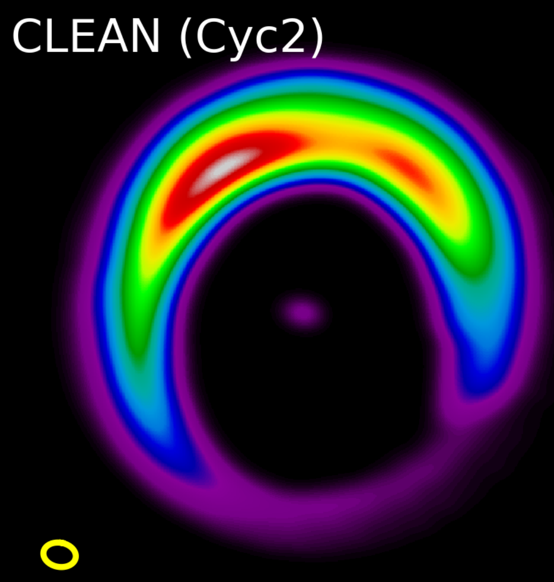
Intermediate config.

Nominal Resolution

Nominal Resolution



Kataoka et al. 2016, ApJ



Fukagawa et al. in prep.

Application to Real Data: Protoplanetary Disks

ALMA Observations of Protoplanetary Disk HD 142527 (345 GHz)

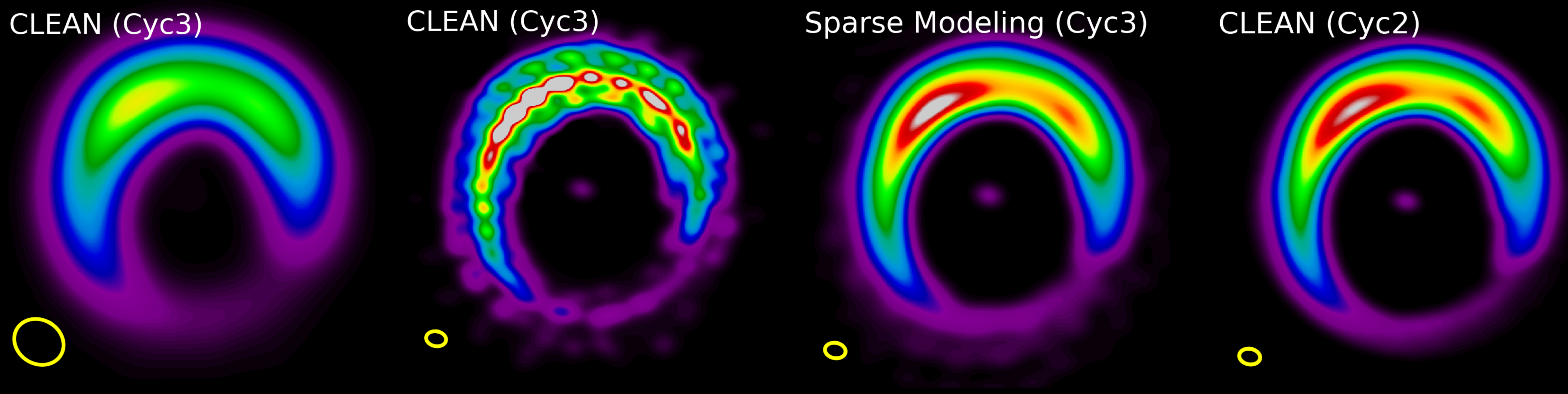
Compact configuration

Intermediate config.

Nominal Resolution

Superresolution (same to the intermediate configuration)

Nominal Resolution



CLEAN (Cyc3)

CLEAN (Cyc3)

Sparse Modeling (Cyc3)

CLEAN (Cyc2)

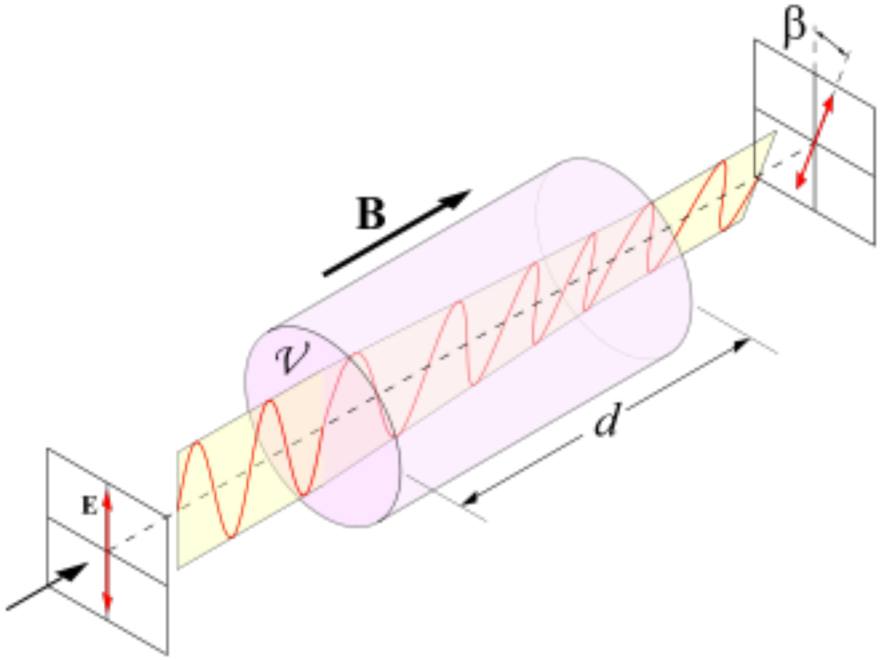
Kataoka et al. 2016, ApJ

Fukagawa et al. in prep.

(Yamaguchi, *KA*, & Kataoka et al. in prep.)

Event Horizon Telescope

Applications to SKA Science: Faraday Tomography



EVPA rotation of radio waves in magnetized plasma

$$\chi = \chi_0 + RM\lambda^2$$

$$RM\ (\mathrm{rad\ m^{-2}}) \approx 811.9 \int \left(\frac{n_e}{\mathrm{cm^{-3}}}\right)\left(\frac{B_{||}}{\mu\mathrm{G}}\right)\left(\frac{dr}{\mathrm{kpc}}\right)$$

Rotation angle is proportional to λ^2
= phase rotation in linear Pol spectrum

This is very similar to what we usually see in interferometric data.

(e.g.) A point source in the image causes a phase rotation in the visibility, which is a spatial spectrum of the image.

$\Delta\varphi = 2\pi x_0 u$ for a point source at $x = x_0$

(x, u) for interferometric imaging; (RM, λ^2) for Faraday Rotation

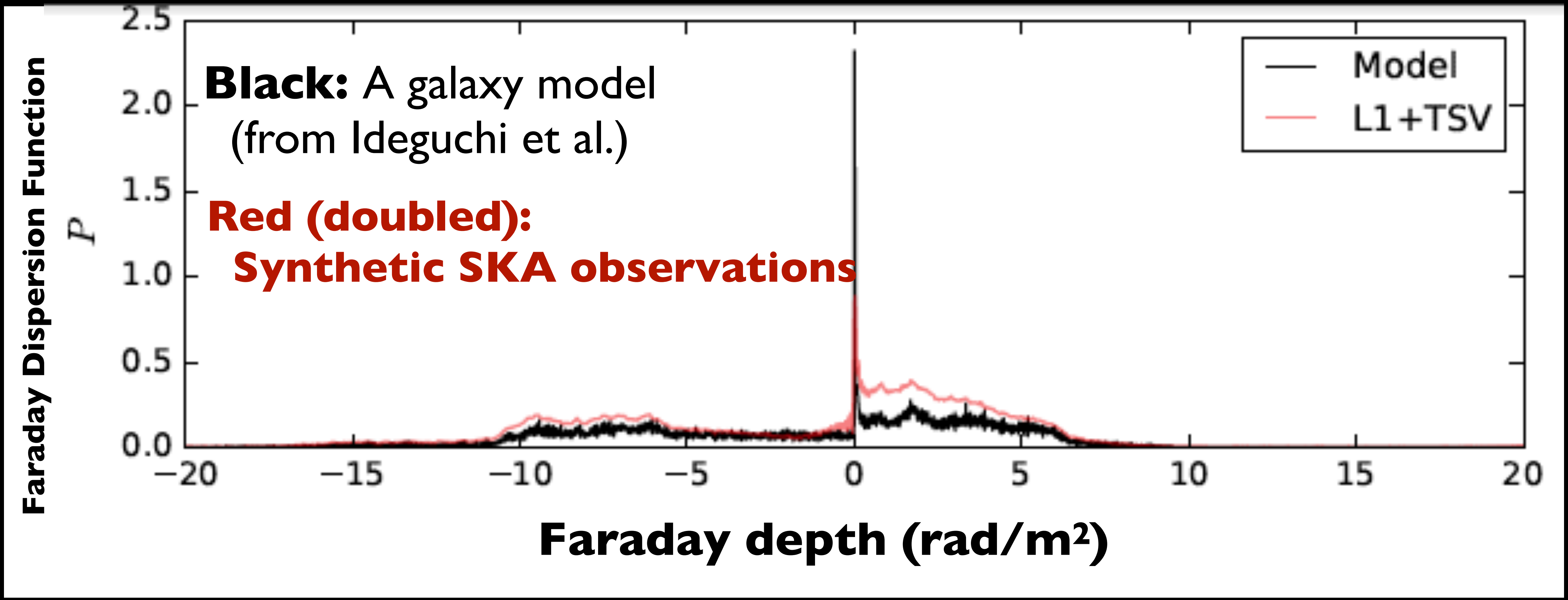
Applications to SKA Science: Faraday Tomography

Y = **A** **X**

Linear Polarization Spectra

Fourier Transform for Faraday depth

Fourier Dispersion Function



(*Akiyama* et al. in prep., Collaboration with SKA-JP Faraday Tomography WG)

EHT Imaging: Fusion of Young Powers & Divergence

Andre Young
(SAO Astronomy)

Kazu Akiyama
(MIT Astronomy)

Julian Rosen
(UGA Mathematics)

Lindy Blackburn
(SAO Astronomy)

Katie Bouman
(MIT Computer Vision)

Andrew Chael
(Harvard Physics)

Michael Johnson
(SAO Astronomy)

Marki Honma
NAOJ
Astronomy

Shiro Ikeda
ISM Statistical
Mathematics

Fumie Tazaki
NAOJ Astronomy

Kazuki Kuramochi
U. Tokyo Astronomy

Challenges for VLBI Imaging

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No good phase calibrators!

We need to carefully CLEAN
so that images are reasonably smooth and sparse, and consistent with closure phases.

Challenges for VLBI Imaging

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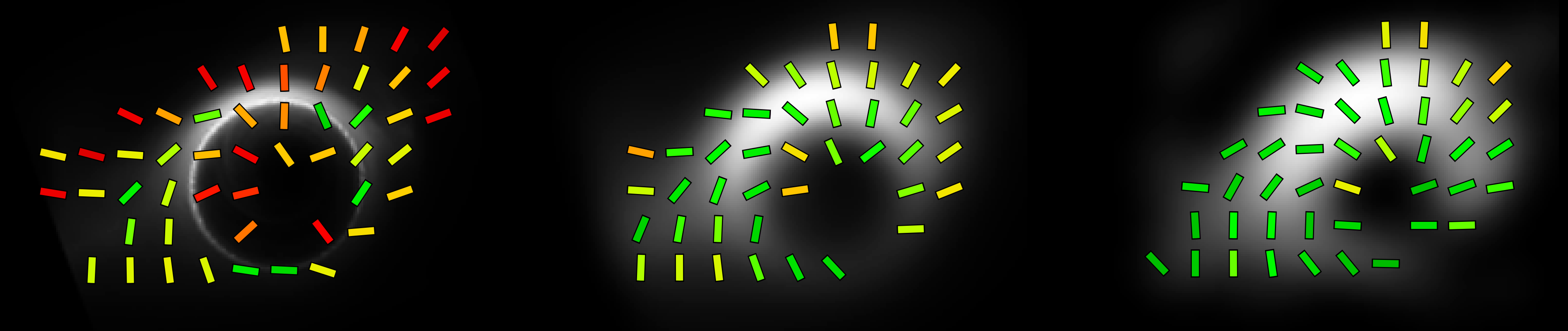
so that images are reasonably smooth and
sparse, and consistent with closure phases.

Solution: Imaging from Amplitudes + Closure Phases



Model

Model (Convolved)

EHT 2017

○ ○

Sparse Modeling: *Akiyama* et al. 2017a, Kuramochi & *KA* et al. 2018
MEM: Lu et al. 2014, 2016, Fish et al. 2016, Chael et al. 2016
CHIRP: Bouman et al. 2016

Challenges for VLBI Imaging

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amplitude gains of ~10-30 %. …., etc….

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so that images are consistent with
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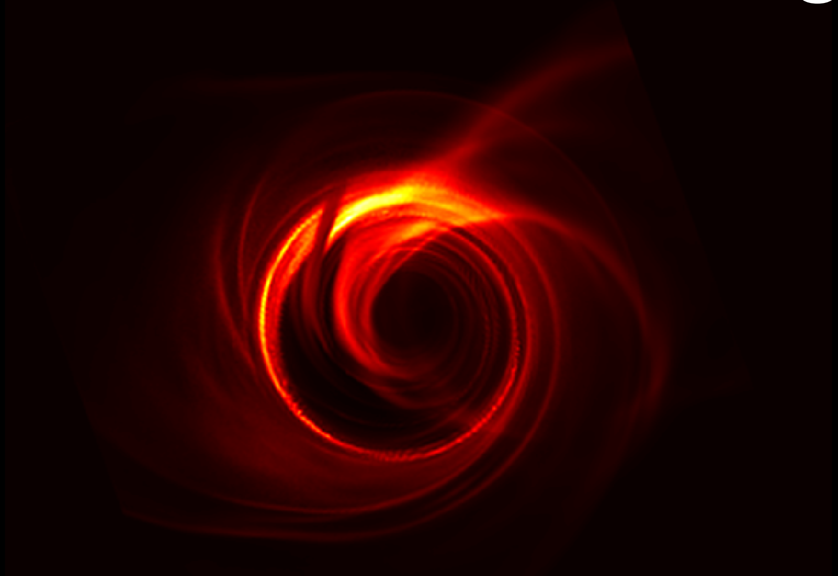
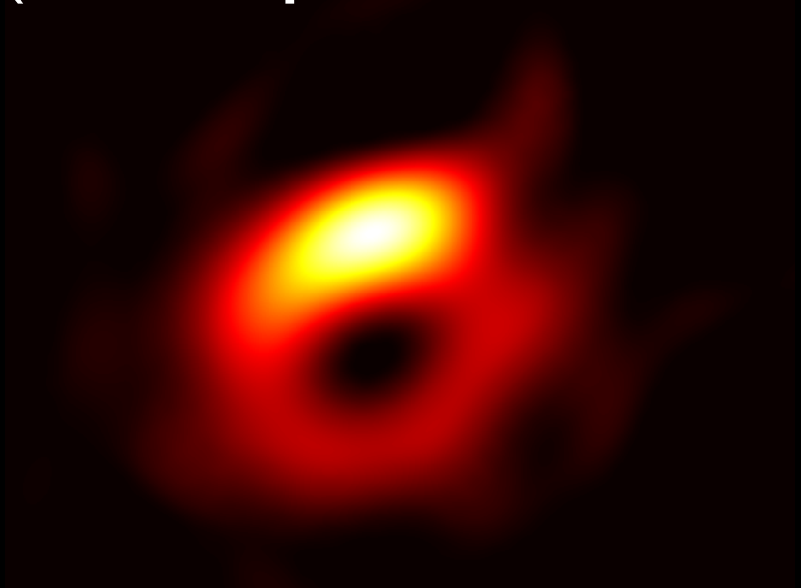
Solution: Full Closure Imaging (Cl. Amplitudes + Cl. Phase)



M87 Jet Model
(Moscibrodzka+17)



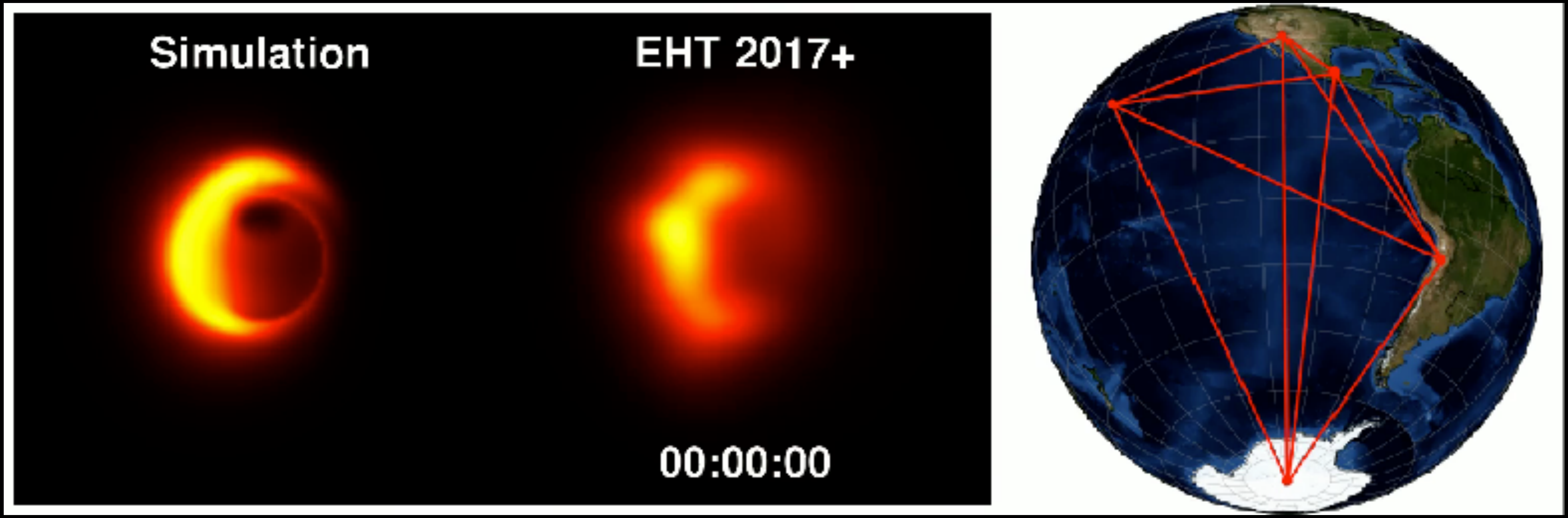
EHT 2017/2018
Full Closure Imaging (Sparse Modeling)

Theoretical Background: Chael , …, *KA* et al. 2018, ApJ, in press.

Challenges for VLBI Imaging

Sgr A* (and M87) has a time variability.

Solution: regularize and solve movies.
(extension of sparse and other regularizers in time direction)



(Johnson ,…., *KA* et al. 2017, ApJ, Bouman et al. 2017, submitted)

Event Horizon Telescope

Challenges for VLBI Imaging

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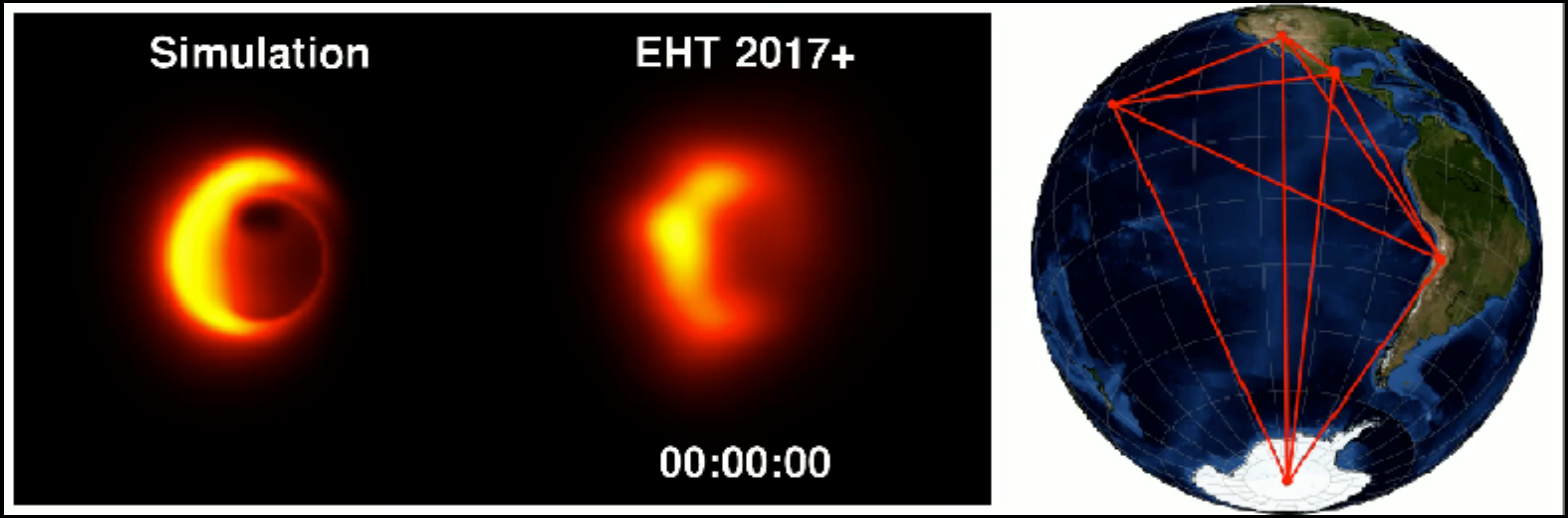
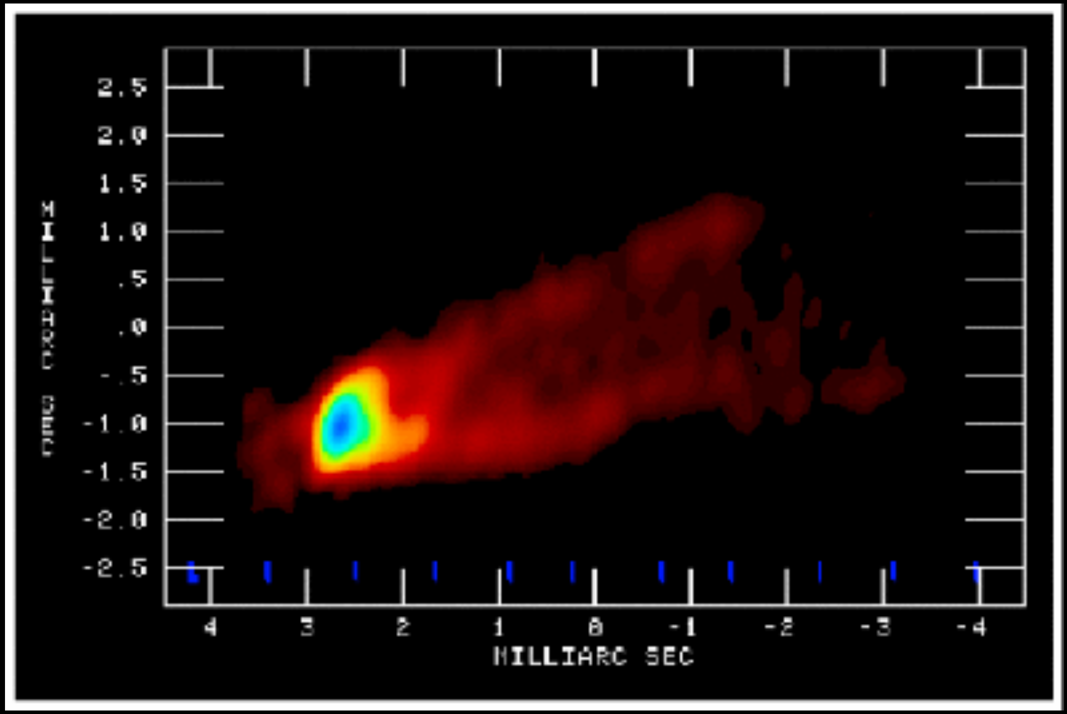
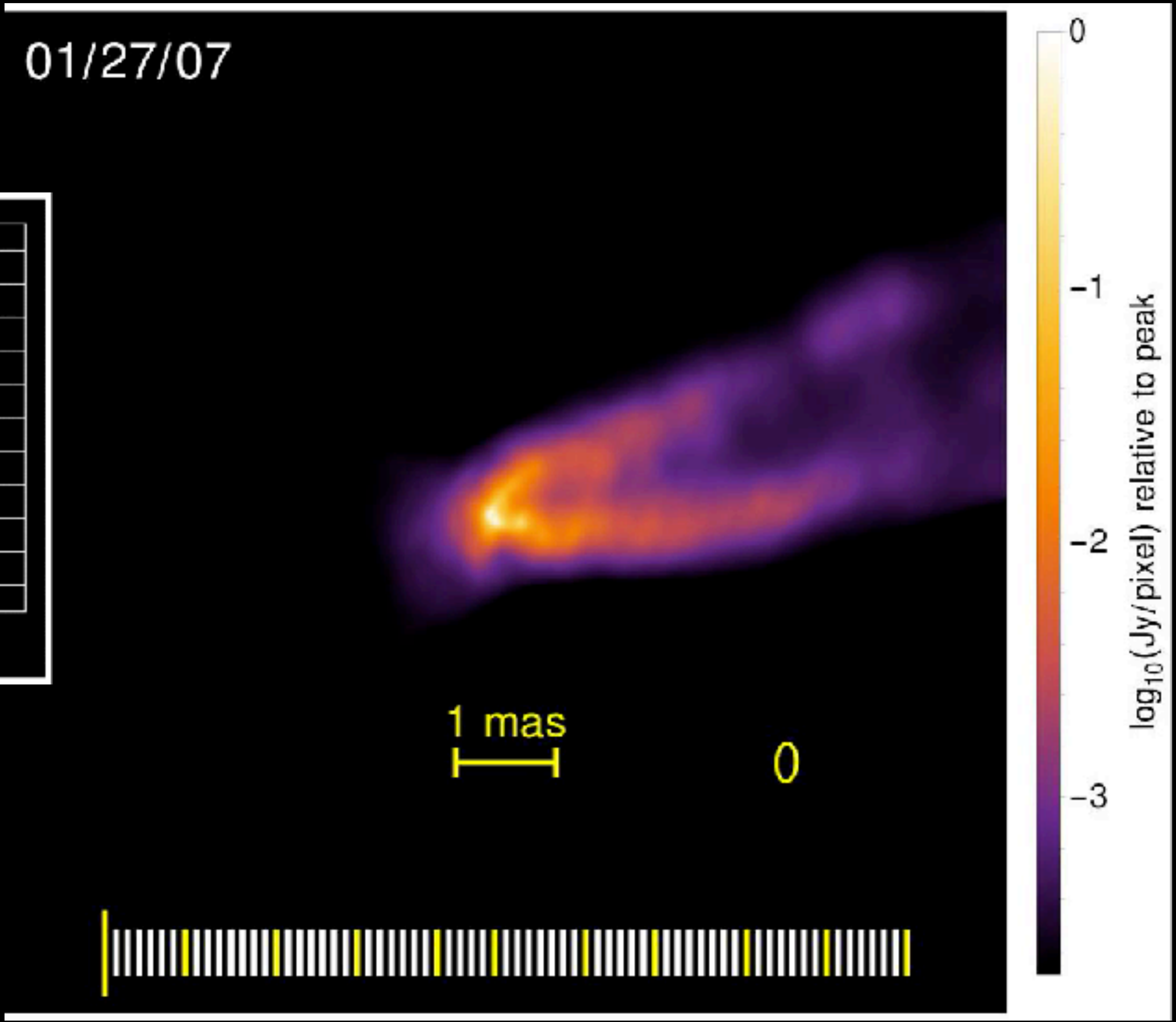


(Johnson ,…., *KA* et al. 2017, ApJ, Bouman et al. 2017, submitted)

Applications of Dynamical Imaging to M87 data

Revision of the
Walker+08 movie





(Johnson, …, *KA* et al. 2017, ApJ)

Implementation for the Next Generation Interferometers

VLBI, Optical interferometers (EHT, VLBA, EVN, EAVN, Radio Astron…)

- Absolute phase information will be lost in general
- Amplitude calibration could be not reliable
- Require closure imaging and self-calibration

Python-interfaced package SMILI (*Akiyama* et al. 2018, to be submitted)

Present & next generation connected interferometers
(ALMA,VLA, SKA, ngVLA, LOFAR, …)

- Should be integrated into 3rd generation calibration packages handling direction-dependent effects (see, e.g., Smirnov 2011, A&A)
- Full complex visibility imaging & selfcal would be enough.

Implementation to CASA (Nakazato, Ikeda, KA, Kosugi, in prep.)

HAYSTACK OBSERVATORY

NRAO

Event Horizon Telescope

Summary

- Sparse Modeling (and other EHT imaging techniques) provide a new opportunity to obtain high-quality, high-resolution images (and movies) from various type of interferometric data sets.

- On-going wide application to various sources and other problems
  - Radio Stars, Protoplanetary disks, Jets
  - Faraday Tomography

- Next Scope: 3D/4D imaging, Wide-field Imaging

HAYSTACK OBSERVATORY NRAO Event Horizon Telescope

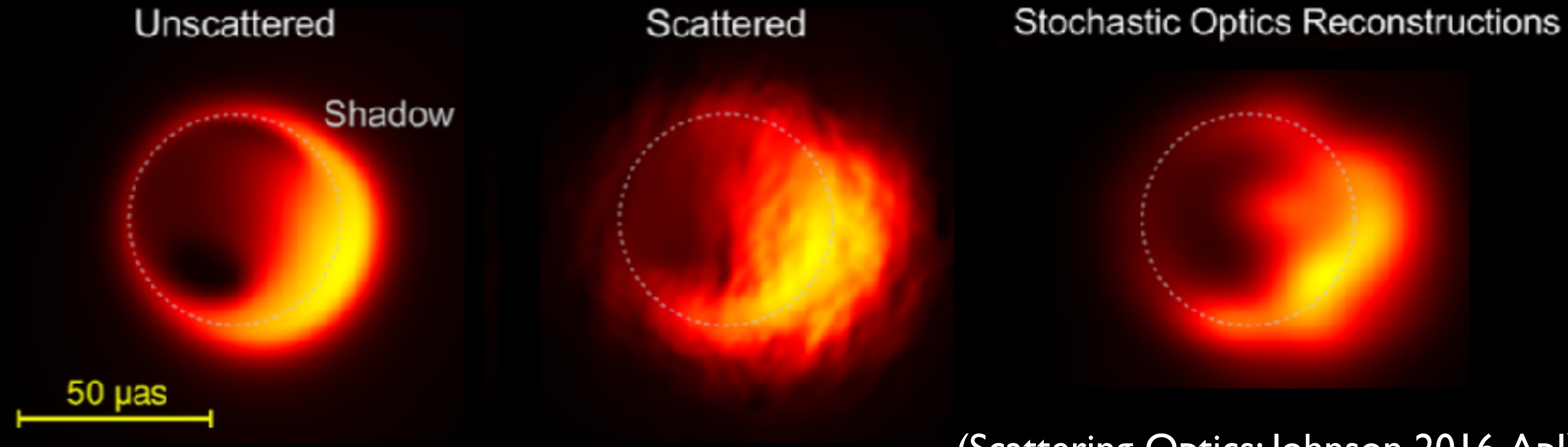
Challenges for VLBI Imaging

Sgr A* is scattered!

Diffractive scattering: invertible

Refractive scattering: not invertible

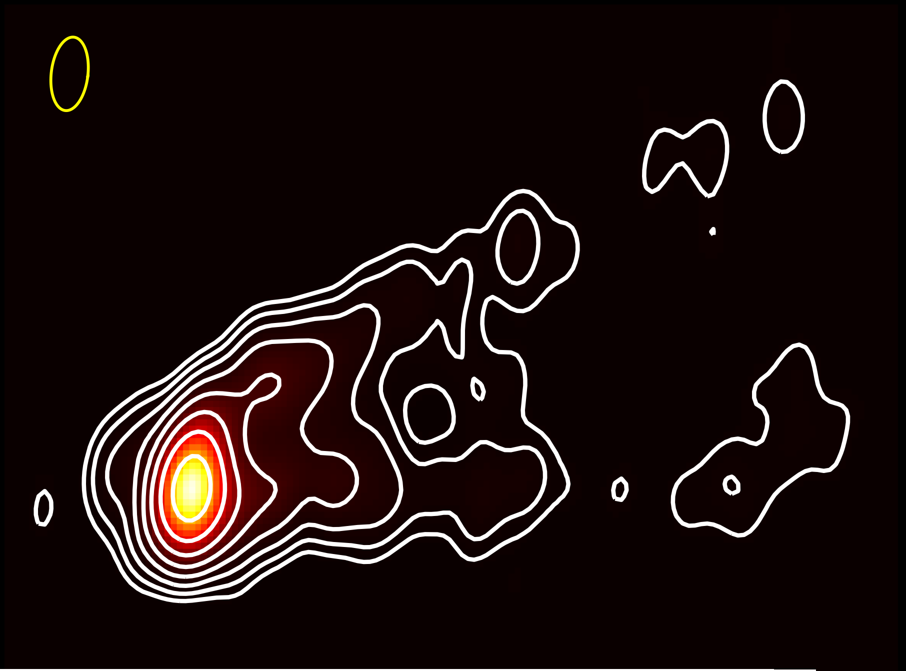
Solution: regularize and solve the phase screen of the refractive scattering as well!



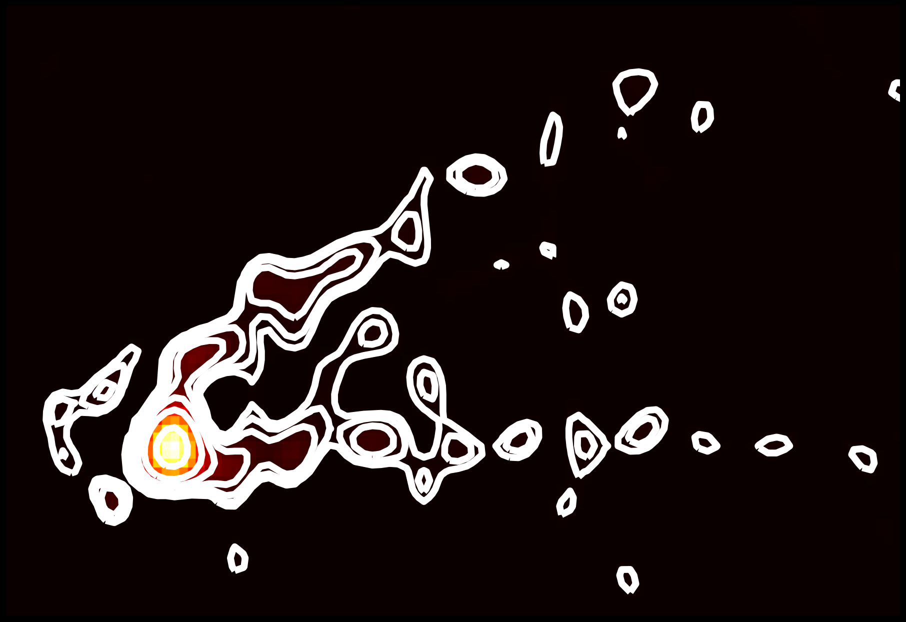
(Scattering Optics: Johnson 2016, ApJ)

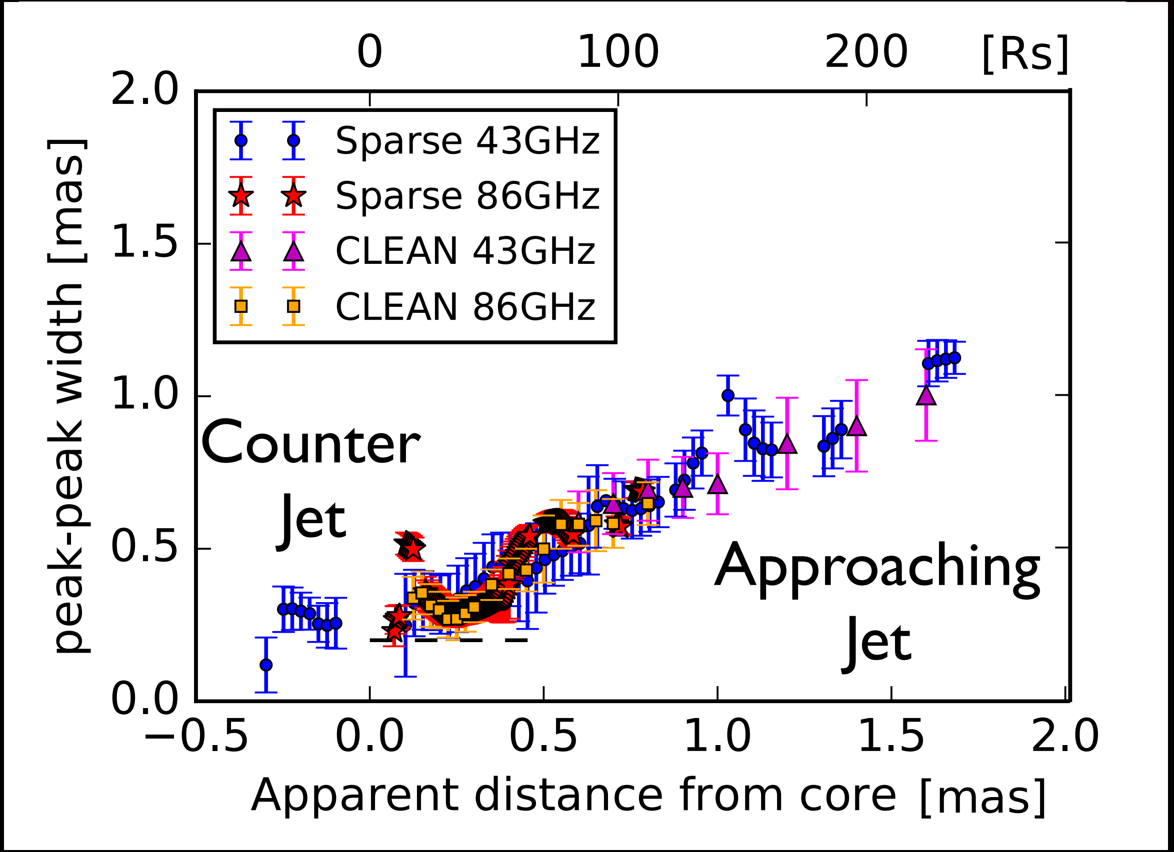
Application to Real Data: VLBA M87 Data

CLEAN
(43 GHz)



Sparse
Modeling
(43 GHz)





Clear reproduction of counter jets

Derived collimation profile of the M87 jet is consistent with 86 GHz data

(Tazaki et al. 2017, submitted)